ABSTRACT: There are circumstances in which we want to predict a series of interrelated events. Faced with such a prediction task, it is natural to consider logically inconsistent predictions to be irrational. However, it is possible to find cases where an inconsistent prediction has higher expected accuracy than any consistent prediction. Predicting tournaments in sports provides a striking example of such a case and I argue that logical consistency should not be a norm of rational predictions in these situations.

Keywords: Rationality, Prediction, Decision Theory, Logical Consistency, Expected Utility, Tournaments

Logical consistency is often assumed to be a necessary condition for rationality. However, when attempting to predict an interrelated set of events, such as a sequence of game outcomes in a sports tournament, there are circumstances where rationality seems to require making inconsistent predictions. Here, I will provide an example where decision-theoretic rationality, in the sense of maximizing expected utility (von Neumann and Morgenstern 2007), conflicts with logical consistency. From a pragmatic perspective, inconsistency may, in some cases, be more rational than consistency.

Every March, the sports world in the United States is abuzz with the NCAA collegiate basketball tournament. The tournament has a simple structure: 64 teams are seeded in a single elimination bracket. The last team standing is the champion. Many fans, both serious and casual, partake in prediction contests. These contests usually charge the participants with the herculean task of correctly predicting, at the outset of the tournament, the winner of every game. The difficulty of this task and the inevitable surprises of the tournament give weight to the tournament’s other name: “March Madness.”

The miniscule chance of a perfect bracket prediction means that the real goal is to give a more accurate prediction than one’s competitors. Suppose you are tasked with such a challenge: generate a complete prediction set that maximizes the expected number of correctly predicted games. Speaking more generally, you must provide a prediction for a sequence of interdependent events and your goal is to maximize the accuracy of your prediction. We will suppose that the relevant measure of accuracy is simply the number of correctly predicted individual events.

For tractability, assume that there are only four teams in the tournament. In the first round, team A will play team B and team C will play team D. The winners of each game will play in the
second round. Thus the task is to predict the outcome of three games prior to any of them being played.

Game 1: team A vs. team B,
Game 2: team C vs. team D,
Game 3: winner of Game 1 vs. winner of Game 2.

Let's make two further suppositions. First, aside from the tournament structure—losing in the first round entails not playing in the second round—suppose the games are independent of one another. Second, suppose you have detailed and accurate knowledge regarding the strengths and weakness of each team. In particular, you can determine the probability of each team defeating any other (Table 1).

Table 1: The probabilities of each team winning against every other. Values are the row team's probability of defeating the corresponding column team.

<table>
<thead>
<tr>
<th>Team A</th>
<th>Team B</th>
<th>Team C</th>
<th>Team D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team A</td>
<td>--</td>
<td>.4</td>
<td>.9</td>
</tr>
<tr>
<td>Team B</td>
<td>.6</td>
<td>--</td>
<td>.5</td>
</tr>
<tr>
<td>Team C</td>
<td>.1</td>
<td>.5</td>
<td>--</td>
</tr>
<tr>
<td>Team D</td>
<td>.15</td>
<td>.55</td>
<td>.55</td>
</tr>
</tbody>
</table>

Note that the win-probability relationships are intransitive. B is likely to win against A, A is likely to win against D, but D is likely to win against B. There may be a number of reasons for this relationship: e.g., perhaps A is very good at exploiting a particular weakness that D has but B does not, even though B has less average talent than D. Anyone who stresses the importance of specific player or positional matchups in team sports should acknowledge such win probabilities as possible. Whatever the cause, such intransitive relationships are not unrealistic in sports. The intransitivity above, however, turns out to have a peculiar effect on the optimal predictions.

Since there are only four teams and three games to be played, it is easy to run through all possible tournament outcomes and calculate which is the most likely. This turns out to be the prediction that B defeats A and D defeats C in the first round and B defeats D in the second round—this is despite D being more likely to win against B if that game occurs. To see this, let Pr(XY) represent the probability that team X wins against team Y and let Pr(X_n) represent the probability that X wins game n. The probability of each team winning games 1 and 2 is known from Table 1. The probability of any team winning the tournament can be calculated as well. D is likely to beat C and is also likely to beat their most likely opponent B. But the probability that B wins the tournament is greater than that of D because B is more likely to reach the second round and D may have to play A.

\[
\Pr(D_3) = \Pr(DC) \times [\Pr(DA) \times \Pr(AB) + \Pr(DB) \times \Pr(BA)],
\]

\[
\Pr(D_3) = .2145.
\]

\[
\Pr(B_3) = \Pr(BA) \times [\Pr(BD) \times \Pr(DC) + \Pr(BC) \times \Pr(CD)],
\]

\[
\Pr(B_3) = .2835.
\]
Between $B_3$ and $D_3$, you should predict the former. However, the most likely winner is actually $A$ because of their high probability to win Game 3, should they win Game 1.

$$\Pr(A_3) = \Pr(AB) \times [\Pr(AD) \times \Pr(DC) + \Pr(AC) \times \Pr(CD)],$$

$$\Pr(A_3) = .349.$$ 

So, it is easy to see that team $A$ is simultaneously the most likely winner of Game 3 and the least likely to reach Game 3. But, what does this mean for predicting the entire tournament? Decision-theoretic rationality demands that you offer a prediction that generates the highest expected utility. In this case, you should maximize the expected number of correctly predicted games. This can be calculated by taking the sum of the probability of a correct prediction on each game. Here is the expected number of correctly predicted games for the best three logically consistent tournament predictions:

1. $(B_1, D_2, B_3): 1.4335$,
2. $(B_1, D_2, D_3): 1.3645$,
3. $(A_1, D_2, A_3): 1.299$.

So among the possible tournament outcomes one should predict $(B_1, D_2, B_3)$. But what if we consider the impossible tournament outcomes? Suppose we were to predict that team $A$ loses Game 1, but wins Game 3. Of course, such an outcome is not possible given the tournament’s structure as winning Game 1 is required to play in Game 3. But, since we are simply trying to maximize the number of correctly predicted games, it is important to consider every prediction we could put forward, including those known to be logically inconsistent with the tournament structure.

In this case, shunning logical consistency enables a prediction that is expected to outperform any consistent prediction. The prediction $(B_1, D_2, A_3)$ is a logical impossibility as $B_1$ contradicts $A_3$—it is equivalent to predicting that team $A$ will both win the final and not reach the final. Despite the inconsistency, however, this prediction yields an expected 1.499 games correctly predicted. This expected return is higher than any logically consistent prediction. The best course of action, according to decision theory, is to predict a sequence of outcomes known to be impossible.

Sport is not the only arena where predictive tasks of this kind may occur. For instance, political analysts frequently make predictions about elections. And, a sequence of elections—e.g. primaries, then general elections in the United States—has a similar tournament-bracket structure, though with more participants in each “game.” A candidate that is unlikely to win a primary election among party supporters may be more likely to appeal to the general population and hence, more likely to win the general election. This means that political victories may be intransitive in the same sense that win-probabilities can be in sporting events. Consequently, in the right circumstances, predicting that a candidate will lose the primary, but win the general election may be the optimal course of action.

A similar tension between rational belief and logical consistency is highlighted by the lottery paradox (Kyburg 1961; Wheeler 2007). Of all tickets in a lottery it seems rational to accept the statement, “This ticket will not win,” and also rational to accept the statement, “Some ticket will win.” Yet, these statements together are inconsistent. The paradox of the preface also reveals a
tension between rational belief and logical consistency (Makinson 1965). It seems rational for an author of a sufficiently long book to state that he believes each sentence of the book to be true, but also write in the preface that the book “undoubtedly contains some errors” (and believe this statement). These paradoxes have led scholars to develop robust accounts of the role of (in)consistency in rational belief, assent and acceptance (see e.g. Hawthorne and Bovens 1999; Pollack 1990). Others have taken these kinds of paradoxes to show that deductive consistency should not be a requirement of rational belief (see e.g. Christensen 2004, Easwaran and Fitelson 2012).

There are, however, a few differences between these paradoxes, as they are traditionally formulated, and the March Madness case above. First, prediction in the March Madness case does not involve any higher-order beliefs as in some formulations of the preface case; the inconsistency results from the specific game predictions and the structure of the tournament. Second, the both the lottery and preface paradoxes are cases where each individual claim is probable, but the conjunction inconsistent. However, the tournament prediction example is a case where the inconsistency is desirable despite some predictions being relatively unlikely. Finally, the central issue of the March Madness case is rational predictions, not rational belief as in the lottery and preface paradoxes.

This last difference is important because, when directing action, prediction may not entail belief. More precisely, the most rational prediction may not always be one we think has a high probability of occurring. For instance, when you are looking for a set of lost keys, the most rational place to begin looking, other things being equal, is the place that you think they are most likely to be. In this sense, you can predict that you left your keys in your office without believing it to be true. The prediction of $A_3$ taken alone seems rational even though we think it is more likely to fail than succeed. The nature of the task prevents us from making the prediction “Not-$A_3$” even if we believe that $A_3$ is less likely to occur than not; to succeed in the task at all, we are forced to make a definite prediction.

Indeed, it is predicting the winner of each game individually without regard to the logical interdependencies that lead to the inconsistent ($B_1, D_2, A_3$) prediction. The prediction for each game seems rational while the aggregate tournament prediction is impossible. However, since the task is to maximize predictive accuracy judged by the number of correctly predicted individual events, it is prudent to neglect logical consistency and aggregate the predictions. If prediction does not require belief, this does not mean we should believe our set of predictions. But, it does mean that we will have to sacrifice consistency to make the best bets. Regardless of one’s response to the lottery and preface paradoxes regarding rational belief, it seems reasonable to say that rational predictions do not require consistency. The case above illustrates an outright conflict between maximizing expected correct predictions and logical consistency. And, when you get partial credit, making inconsistent predictions can be the most rational course of action.

What does this mean about how we should approach prediction and analysis of sports? Admitting the possibility of intransitive win relationships should inform our attitude about tournament winners. Did the champions win because they were a great team, or did they merely have a lucky position in the bracket where their specific strengths matched up well against their particular set of opponents? Furthermore, if real sports teams can have intransitive win relationships amongst one another, it must be admitted that many common methods of ranking teams are missing an important element. For example, the Elo rating system (Elo 2008), originally used to rank chess players and now applied to several major sports (see e.g. Silver and Fischer-Baum 2015), only considers an overall aggregate performance level and does not
consider specific strengths and weakness. As a consequence, such rating systems produce neither intransitive rankings nor inconsistent tournament predictions.

More detailed methods of comparing teams, such as a player-vs-player comparison as is sometimes done when analyzing batter-pitcher matchups in baseball, may be able to identify particular strengths and weakness of a given team compared to another. This second kind of method may allow for predicting intransitive win relationships and thereby allow for inconsistent predictions when applied to whole tournaments. Such inconsistent predictions should be viewed as a feature, not a bug, of these more detail-oriented approaches. However, the loss of transitivity does mean that there may be simply no objective way to rank a group of teams, just as there is no best move in Rock-Paper-Scissors.

Finally, if you find yourself completing a prediction bracket next March, unless the rules of your specific contest dictate that you submit a logically consistent bracket, there is good reason to consider the logically inconsistent alternatives. Given the size of the tournament and the fact that most scoring rules count later games more heavily than early games means there are many potential places for inconsistent predictions to generate higher expected scores. If you care about winning the contest, abiding by logical consistency may well be irrational.

Acknowledgements:
I would like to thank John Basl, Patrick Forber, Ron Sandler and Kevin Zollman for comments and discussion on previous drafts of this paper. I am also grateful to John Russell for helpful remarks in revising the paper.

REFERENCES